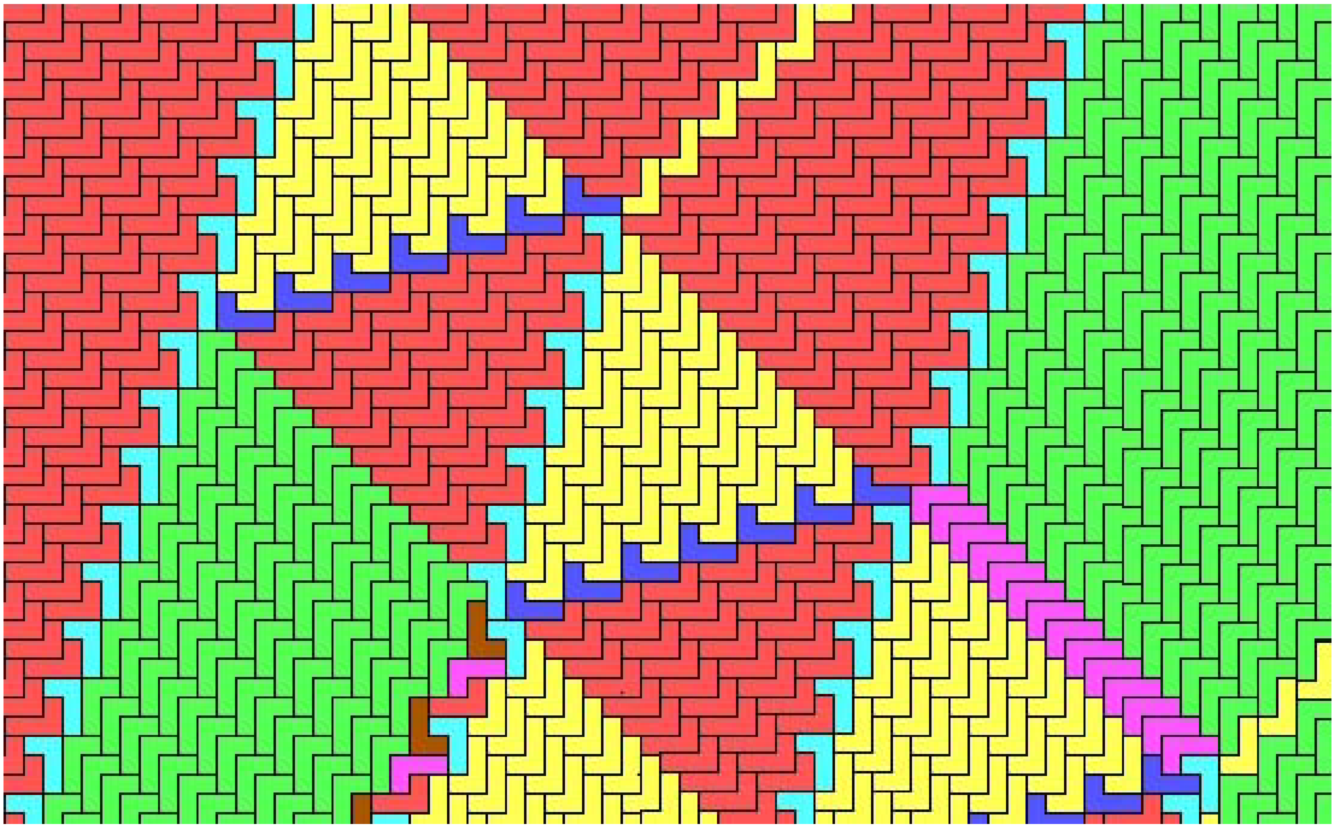
The Fruits of Asymmetry



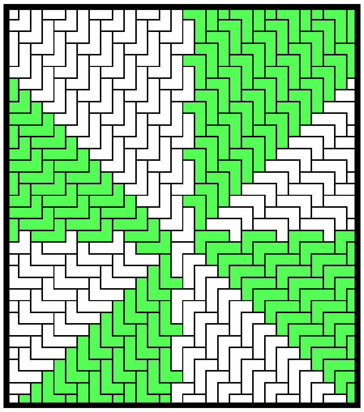
Here is an old tiling of mine which regularly crops up when I am perusing this section of my work on Tessellations (usually looking for something quite different), yet which each time I come across it, invariably waylays me with the inevitable new questions that it poses.
It was not, when I created it, the result of any significant period of directed research, but, on the contrary, was the culmination of a rather easy-going wander through a pleasing corner of odd tilings, which seemed to deliver a great deal more than it initially promised. It was not that I wasn't delighted when I found it. I most certainly was, but I did not endow it with anything other than the pleasure of its construction.
It just did not seem to be a significant piece of work at all.

I was not to know that it would lead, every time that I alighted upon it, to developments as wide apart as formal models of DNA to those for semi-permeable membranes, and even a new means of Realisation of Form using only colour cycling.
So, it is long overdue for a much closer look at this fruitful fragment under the title of ***The Fruits of Asymmetry***.

Have a close look at the tiling shown above.
It is one of many hundreds that I have developed using re-entrant polygons (in this case using a singly re-entrant asymmetric hexagon – in other words an **L** shape). There is an extensive variety of such basic units, but this one has promised an almost infinite number of different and interesting tilings (tessellations), and I keep returning to it, for it encapsulates in one picture a crucial set of tiling types and tricks, which seem developable in many different ways.

Earlier investigations into what I called **Families of Tessellations** of this single basic unit had revealed initially a very boring and simple set of eight tilings, involving the four different orientations of the unit, plus their mirror images.

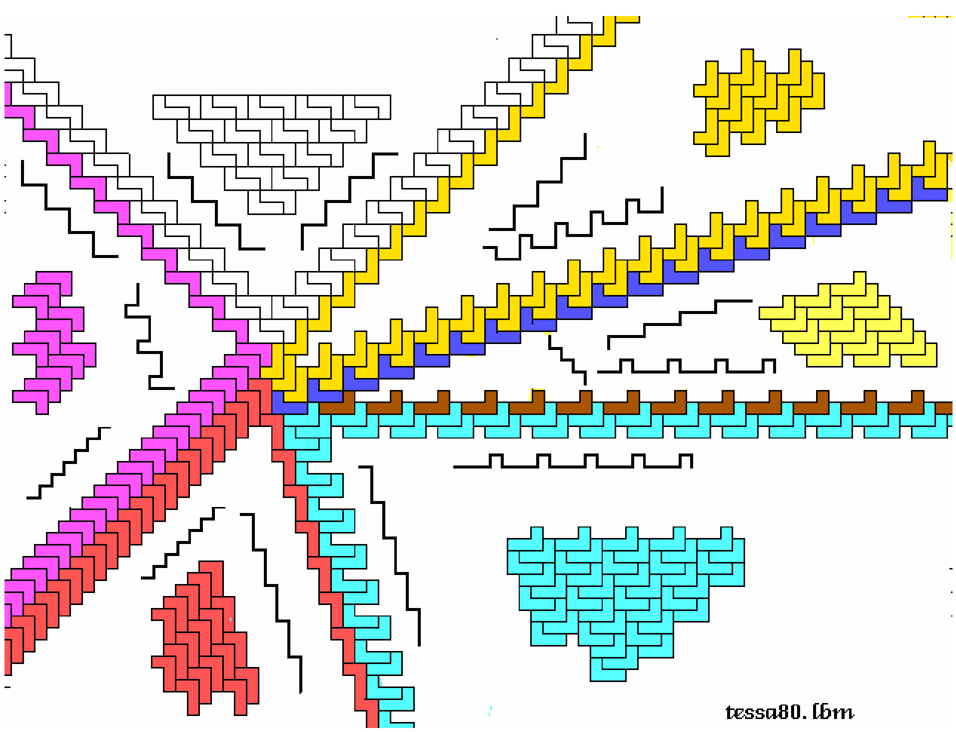
These could abut, without any gaps, along very simple **"staircase" boundaries** (a couple of which occur in this current picture), which were usually infinite in extent.



But, by the time I had got to making this particular figure, areas (or patches) had been found to be possible, and the crucial investigation switched to "amenable boundaries" and even "nodes" (where several tiling patches came together seamlessly).

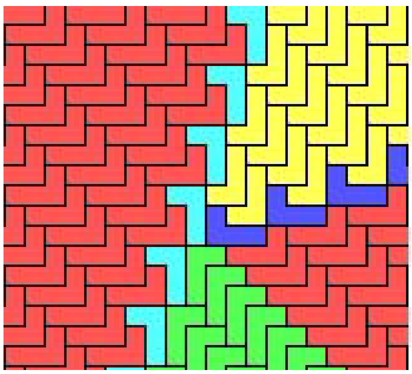
As can be seen, all sorts of boundary features were discovered, which became necessary to allow such a mix of tilings to both co-exist and define various areas, and these could (it turned out) be at many **different angles**. Five boundaries are included here, but some present the same angles, while others do not, and the question arises about how many different boundary angles are actually possible within this Family alone.

Before I address such a question, a more general muse seems in order.
Approaching the finding of tessellations of the singly re-entrant asymmetric hexagon (the **L** shape) in a new way generates a surprising sequence. The method is to look ***primarily*** at achievable boundaries, and in particular, at the **angles** that they present. Though these boundaries are obviously "steppy", the found tessellations do present opposite sides which tessellate with themselves (and often with other different tilings too).
The approach, therefore, initially looked for just such staircase-like boundaries, which could be characterised by the relative lengths of each **Step & Rise** of the staircase. Obvious ones such as **1:1, 1:2, 1:3** etc. could also be added to with more complex and less linear with boundaries such as **2:3, 3:4, 2:5, 3:5** etc...



Perhaps surprisingly, many of these were very easily found, but not always as “single unit based repeats”. They often required what I termed “higher order units”, but not usually more than one higher order units” per tiling. Often mixes of multi-unit forms were necessary.

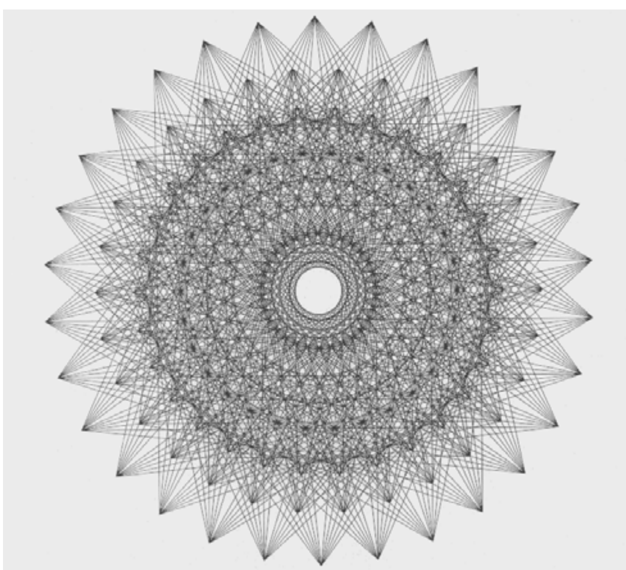
Clearly, once these have been established, the next step would have to be finding the various “different” tilings which presented the ***same boundary form***, then these would at least tessellate perfectly with one another via a single, though infinite, boundary.



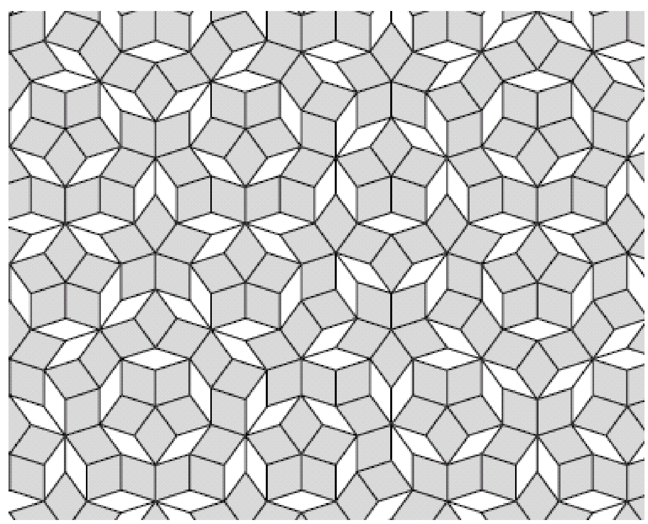
A Node

The next question would be “articulation” – where that boundary, instead of continuing infinitely, ***ended*** in some kind of **Node**, and seamlessly abutted with other tilings of the same Family.

Elsewhere, I have massive amounts of such stuff, and very early on I was in need of assistance, and sought the experts in the field. Wherever I looked at that time, the names Coxeter and Penrose came up, and I immediately got hold of the former’s master tome. He seemed only to be extending formal considerations into “n” dimensions and tilings (or tessellations), which would be achieved using “Polytopes” (rather than polygons or polyhedrons).



A Coxeter Polytope



A Penrose Tiling

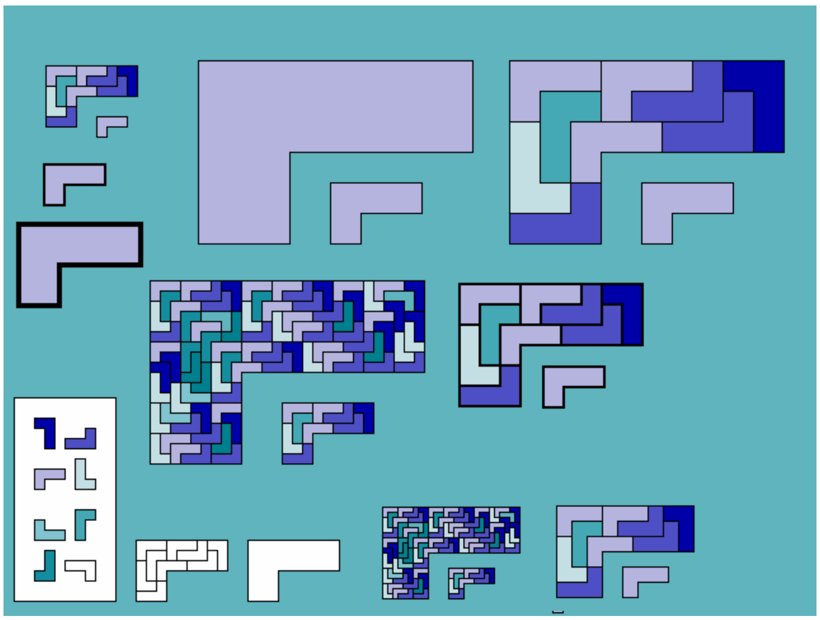
But what I was presented with there was nothing like what I was investigating. Indeed, he ***always*** assumed maximum symmetry, and soon left behind my kind of earth-based investigations, for “much more interesting” offerings provided by multi dimensional Universes beyond my limited objectives. (See his higher dimensional polytope shown above)
No significant help was available there!

And though Penrose did indeed sometimes use re-entrant forms, his whole line was concerned with **near-symmetry cases** (see his image above), and again, his work was no help either.

So, I had to continue alone!

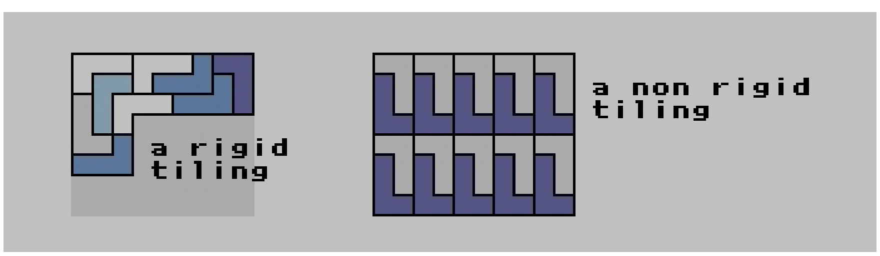
What was evident to me was that the essential component in these formal investigations was Symmetry!
While my colleagues in Physics were totally wedded to complete maximal Symmetry (and indeed Super Symmetry), I was finding that instead of the limited numbers of tilings and crystal forms that were available to them, I was finding (in 2D at least) a seeming infinite complexity of Forms.

With **maximally symmetric** units, these all funnelled down into a minimal set, but with re-entrant forms (the first deviation from maximal symmetry), and various regular asymmetries, the possibilities were vastly increased.
Indeed, from monolithic tilings (using a single basic unit in a single way) there occurred a wide range of hierarchical forms, which could be different or ***mathematically*** similar at each succeeding level.



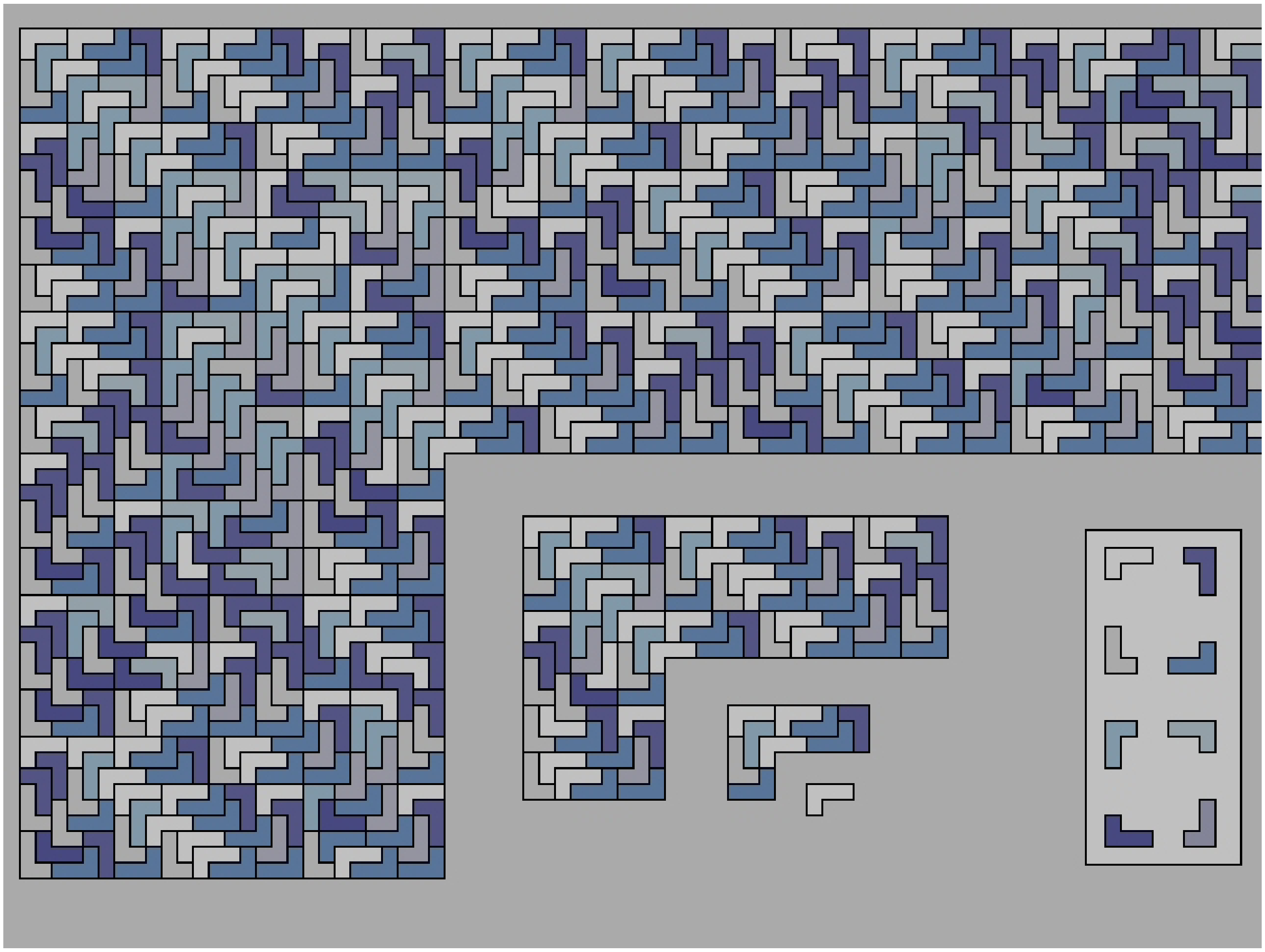
Many new qualities, which were NOT available with maximally symmetric units, emerged with my asymmetric forms. Families of tilings which could "abut" perfectly with one another appeared, and characteristics such as "cleavage lines and planes" became possible.

Tilings could be constructed which were totally rigid and locked, while others could be easily formed which displayed many different "fault-lines", where wholesale movements of one area with respect to others were clearly possible.



Rigid and Non rigid tilings

Hierarchies with dissimilar super-units (macros) could also be easily formed, and some soon seemed indistinguishable from "random mixes" (see image show below)
I even had to invent a colour cycling based system of colouring in the various units with a strict sequence of related colours, so that when these were automatically cycled (in animations and using a finite palette) the various units seemed to "flow"(though they were, of course, totally stationary).
What were revealed by this means were the various hierarchical super-units (Macros), out of the seemingly chaotic orientations, of both basic units and Macros.



What this formal research delivered was also about Reality, and the vast increase in **formal patterns** available once a departure had from maximal symmetry had been allowed.

It was clear that the underlying reason for the possibilities in organic chemistry was that Carbon allowed increasingly complex chains, branches and even rings, of basic units, which ***always*** delivered re-entrant molecules.

> NOTE: Now, a critic might immediately object that my work was **first**, only two dimensional, and **second** using only right angles. But they are the obvious analogue of tetrahedral forms in 2D. I have absolutely NO DOUBT that with similar investigations into three dimensions using forms based on the tetrahedron similar things will be found. In spite of its obvious limitations, Mathematics does have the most important property of **Universality**. Form is Form however many dimensions are involved!

The possibilities in such substances could not be predicated merely on their component elements or even their relative proportions. Substances with the exact **same** proportions of basic elements could occur as quite different substances and with very different properties.

The real biggy in all of this had to be Life!

I always remember the revelation that you could have crystalline forms of living viruses!

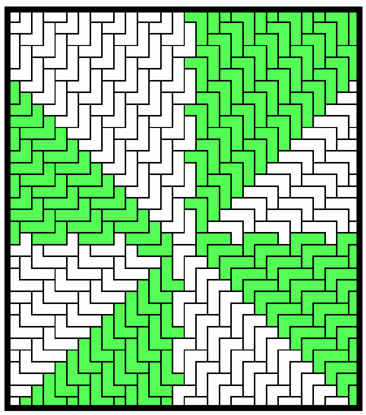
Surely, that was impossible, as living things would not have the usual units that occur in crystalline forms? They would almost certainly be re-entrant, wouldn't they?

Of course, though I romped along the evident formal path of investigating re-entrant units in tessellations, which was because I realised what could be done. I was NOT because I expected any results that I found would in any way play a role in explaining Life!

It was a new line of evidence, which along with other, much more important factors, would, in an **Emergence**, precipitate the first appearance on Earth of the initial Life Forms.
I would, quite obviously, not be in a position to deliver that by purely formal contributions alone.

Now, finally returning to my initial illustrated figure, and my musing about orientations of inter-tiling boundaries, using singly re-entrant asymmetrical hexagons (the **L** shape), I had, fairly quickly, found 19 different possible boundary orientations. In addition I had also revealed how **Nodes** allowed finite terminations of such boundaries and hence the division of the plane into various areas (or patches)

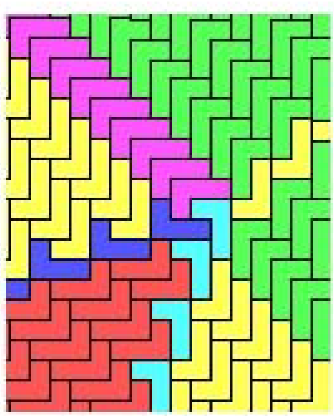
But the area will remain narrow and unrevealing if we only allow ***familiar*** types of tiling which abut with one another directly and without difficulty. Indeed, the whole set of these can be demonstrated on a single diagram shown below.

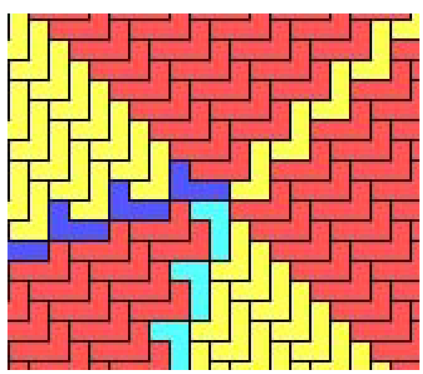


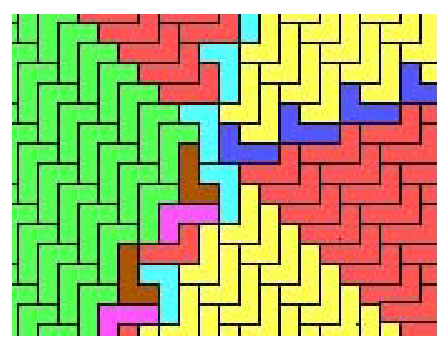
But, once we also allow ***accommodating*** "Strings" and "Braids" between adjacent tilings, to them fit, the possibilities are vastly expanded.

NOTE: Though perhaps stretching it a bit far, the example of DNA comes to mind, which are folded into quite small volumes by the positioning of **Histones** adjacent to particular points along the ,molecule. These do nothing other than make the shape both compact and "geographical" – imposing unique patterns on the periphery of these molecules and facilitating all sorts of processes, which would be impossible if the molecules were stretched out into one long linear form.

When these possibilities are studied, the terminations of boundaries produce a series of very different Nodes, where the single elements of the Strings or Braids come together to accommodate all converging boundaries.
In the diagram below four characteristic Nodes can be seen.







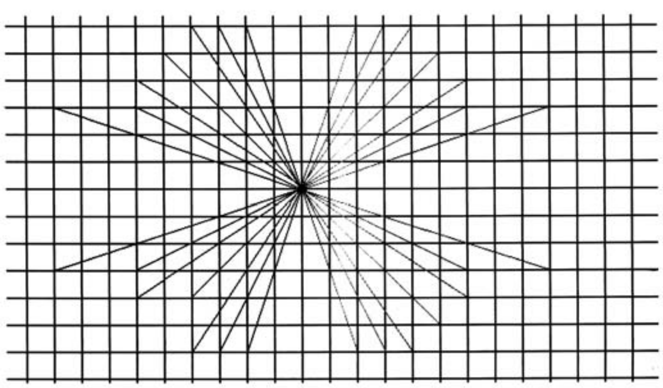
Each brings together up to five tessellations, via Strings or Braids as boundaries, into the focus of a **Node.**
In each of these, the elements of the contributing Strings etc. fuse into Nodes which seamlessly accommodate all the participating elements. When looked at from the point of view of "accommodators", rather than tessellations, the diagram becomes a network of Strings and Braids connected by nodes, wherein the main elements are from the Strings etc..
You are forced to consider crystals and "inclusions" from a similar point of view. The Strings are always somewhat "foreign" to the units making up the tilings, and hence "gather" in boundary regions and nodes – almost like deposits in the rocks of minerals and metal ores.
Of course, to make the tail wag the dog, and see such natural phenomena ***entirely*** in terms of the forms alone would clearly be incorrect. But there is no doubting that, in mixes of participating elements, various formal arrangements predominate – from the "obvious" tilings to infills and flaws of "foreigners" in the final arrangements. (More about such things will be dealt with presently)

Let us return to the Angles of Boundaries.
From a handful of obvious tessellate-able boundaries a bit of research with this unit rapidly generates a surprisingly large number of alternatives.
If we look into such boundaries as having “staircase-like forms, and allowing, in addition, a bit of forwards and backwards variations, we can categorise them all (as mentioned earlier) as having “gradients”. The simplest would be a “1 in 1 “gradient (labelled as 1:1) and delivering a 45^{o} angle to the horizontal (plus its mirror image 135^{o}).
Clearly many others can be very easily constructed, but for them to be “tessellate-able”, two different forms with the same (though mirrored) characteristics will be required.
Let us show a few and begin a list.

slope					
1:1	**45^{o}**	**45^{o}**	**135^{o}**	**135^{o}**	
1:2	**30^{o}**	**60^{o}**	**120^{o}**	**150^{o}**	
1:3	**22.5**	**67.5^{o}**	**112.5**	**157.5^{o}**	
1:4	**18^{o}**	**72^{o}**	**108^{o}**	**162^{o}**	
1:5	**15^{o}**	**75^{o}**	**105^{o}**	**165^{o}**	
1:6	**12.5^{o}**	**77.5^{o}**	**102.5^{o}**	**167.5^{o}**	
1:7	**11.25^{o}**	**78.75^{o}**	**101.25^{o}**	**168.75^{o}**	

And, of course our “one step back, two steps forward” versions will gives us other gradients too
and many more. But if we merely present these on a diagram to show possible “gradients”, the richness of achievable boundary gradients becomes immediately evident.

slope					
3:5	**33.75^{o}**	**56.25^{o}**	**123.75^{o}**	**$^{o}146.25^{o}$**	
2:3	**36^{o}**	**54^{o}**	**126^{o}**	**144^{o}**	
3:4	**38.57**	**51.43^{o}**	**128.57^{o}**	**141.5^{o}**	



Of course, it is when many different boundaries can occur in a **single overall tessellation** of various abutting individual tilings that difficulties can occur. This is because to include such a number of different tilings means that all the boundaries must terminate. These boundaries are normally infinite, so to become finite, they must end at what we call **Nodes**, and once more, our initially illustrated example delivers several of these.

Flaws

Taken to the limit, asymmetry in tilings must arrive at **patches** with **no** evident pattern – and these we term **Flaws**. A Flaw is a "patch" without pattern (or to be more exact – without discernable pattern, with no evident repeats)

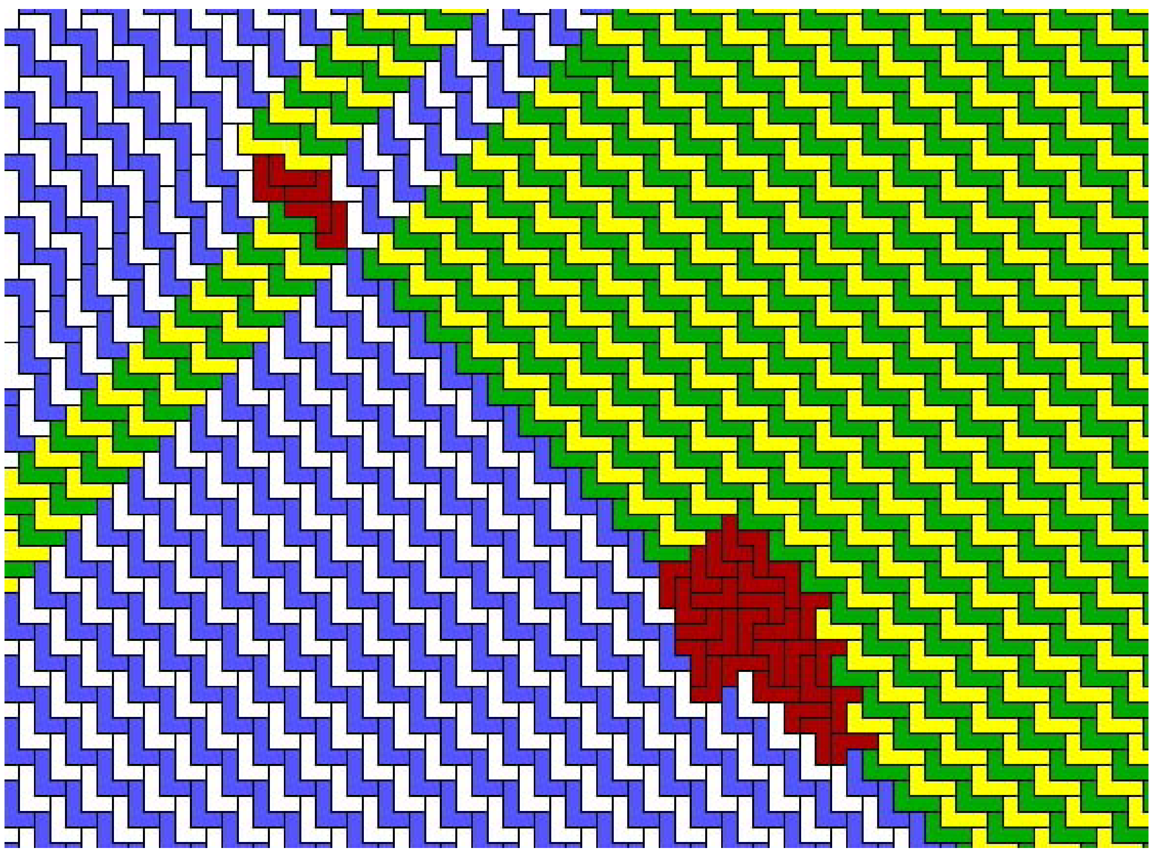
And it is easy to start any attempt to randomly fill a space with our chosen asymmetric units, only to find that you have, by the very method of construction, ensured that gaps will irretrievably be left, and hence **no** overall tiling will be possible.

So, even in a randomly organised "patch" there has to be extendibility, and a "policy" of not leaving the wrong sort of edges has to be found and then rigorously followed.

Yet patches of random, un-patterned tiling can be included within extended tilings, and without leaving any gaps, as Flaws.

Included in the accompanying diagrams are two such Flaws. Now, though these do not by any means exhaust the possibilities, these two Flaws do represent two very different yet significant types.

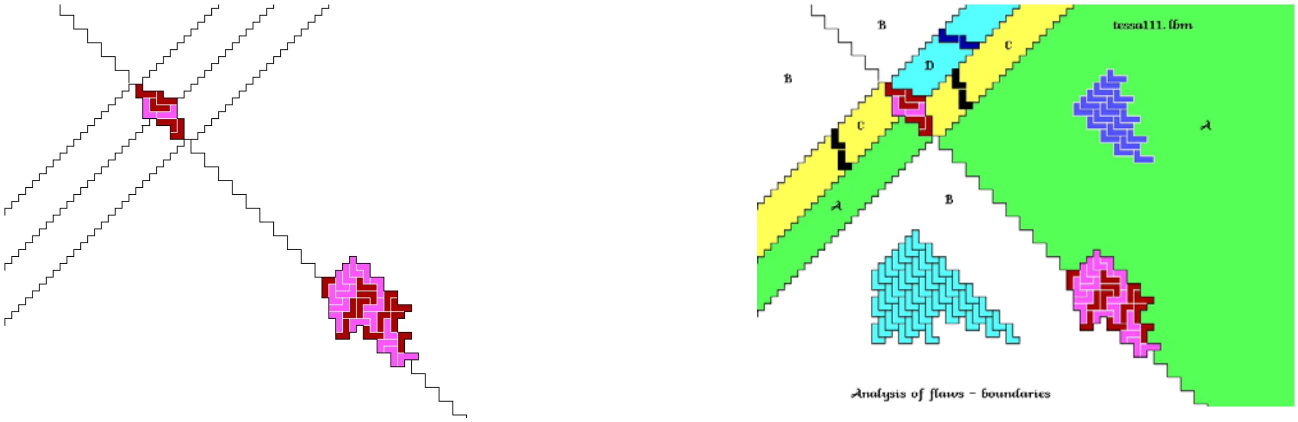
The first (***upper left***) is what I term a **Seed Flaw**. It is best to think of such a patch "occurring first", and only afterwards is surrounded by coherent tilings.



In this case, its outline does not allow a single surrounding tiling. For if that were the case, the obvious form of the patch would be identical to that of the surrounding pattern – it wouldn't then be a Flaw, but merely a patch of the surrounding tiling.

Here we do have a Flaw and if affects what tilings can abut to it. In this case **eight** tiling areas surround the Flaw, and consequently these are only possible with **eight** boundaries propagating outwards from the flaw.

The Flaw sets the subsequent patterns of tilings – hence its appellation as a Seed Flaw.



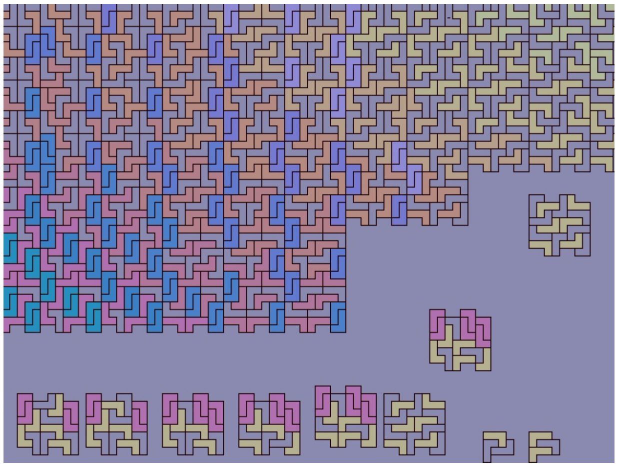
Analysis Diagrams for the Two Types of Flaw

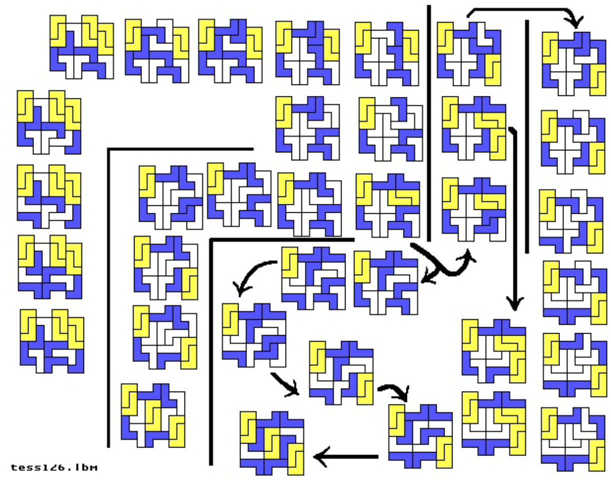
The second Flaw (***lower right***) I term an **Infill Flaw**.
Here we can conceive of it occurring in a different way. We here consider that prior totally tessellation patterns occurred first, and are gradually filling the available space, though not so much as a Tide, but in many directions simultaneously. But such a process could easily leave a hole (yet to be filled) in which some stray units NOT part of the overall tiling become fixed.
To enable the total filling of this space, subsequent added units could also not conform, so the patch gradually got filled with an incoherent mix of units.
NOTE: Now much later in my research I did manage to make a patch entirely surrounded by a single coherent tessellation, which therefore did NOT have any necessary boundaries (as there is in this example., but it was using a different unit and took a concentrated effort precisely to come up with such a patch and context to find it.
Though this is a purely formal investigation, it does throw light on Symmetry and Asymmetry generally.

Symmetry, in tilings taken to the limit, can produce a single monolithic, coherent tiling, but such a form is too ordered to permit any variety. At the other extreme, we can conceive of a situation so lacking in any coherence, that no real tiling is achieved and gaps abound.
This also gives no chance of anything of interest happening.

But Flaws, Families, Boundaries and Nodes do indeed open up many new possibilities.
Islands of conformity (single, coherent tessellations) can be bordered by Strings and Braids, or even themselves surround Flaws, which, in turn, cam affect what happens thereafter.
The mix of pattern and lack of pattern – of Order and Chaos, seems to offer almost endless possibilities.
I even discovered what I termed a "***jigsaw piece***" built from our asymmetric L unit, which I found had a very large number of varieties, and which generated an overall pattern, where every single added jigsaw piece was only slightly different from those immediately adjacent to it. The resulting pattern presented an "evolution" of these jigsaw forms, and even showed branching to alternative paths (as shown below).

Now all this is, of course, ONLY Form!
It is not physical causality, so we must not let such a **formal tail** wag the ***real dog***, as many mathematicians are prone to do.
It doesn't ***explain*** things in the real world at all.
But it does reveal a side which is bound to emerge. The usual mistake is to make the Form ***the cause*** for things in the real world, which is, of course, total nonsense.





This is the Evolutionary Jigsaw Tiling

This is part of the Analysis

Form is always the formal path that can result from a particular nexus of physical causes. We can **describe** such forms and even ***predict*** outcomes, knowing the Form, but that does NOT mean that we can ***explain*** why things happen the way that they do. To do that, we have to go beyond Form, and beyond mathematics and into Science.

Nodes

Though areas can be formed using only normal, direct boundaries between tessellations, they tend to be somewhat restricted, and it is only when Strings and Braids are interposed between tilings that the possibilities are greatly increased.

What happens, however, where several (3 or more) tilings must terminate, is that a special "kind of flaw" interposes, which is best renamed as a **Node**. Four of these have been extracted from our originally included figure, and this allows us to consider their nature in more detail.

The first thing we notice is that these Nodes are NOT randomly constructed at all. They are actually made almost exclusively out of the units of the Strings or Braids which separate the various tilings involved. It is a collection of these units which always comprise every Node. And this should be what we expect! After all the Strings and Braids are what make the tilings fit together, so we should NOT be surprised when elements from the added boundaries comprise the unifying Nodes too.

But note that such a "Macro Unit" **cannot** itself ***tessellate*** alone to fill any areas, and this is why a Node MUST be classified as a special kind of Flaw. The Node for a given context effectively presents the maximum possibilities for other Strings, Braids and tilings to tessellate with.

It is worth seeing how two strings on either side of a narrowing tiling patch come together to finally terminate it.

And, as we go around a Node, we see the same sort of thing for each tiling in turn.

Stepping back somewhat from a picture such as the initial one in this paper, we see that the areas of tessellation are rigid and fixed while the "possibilities" presented at their boundaries by the Strings, Braids and Nodes are diverse and extend into various kinds of Flaws too.

A single monolithic tiling or a complete jumble (taken as its opposite) do not seem to offer much. Both seem static and unlikely to lead to anything new as they are extended. But boundaries present the real areas where the "new" can and do occur!

Some Comments on this Paper

To undertake such a muse as this about Asymmetry, purely in formal terms, may seem to be a pointless exercise, but I would contend that it is not.

At this time in the history of Science the whole of the Sub-Atomic Physics community subscribes to a position that is clearly the exact opposite to the points made above, with their now almost universally accepted Super Symmetry ideas.

I would contend that the assumptions that brought them to this position are "not of this World", and inhabit purely formal systems, drained of all but formal considerations in Ideality – the World of Pure Form alone.

In contrast, Asymmetry is no formal organising principle, but a reflection of the real nature of Reality. Indeed, it is never from Symmetry that the New ever emerges, but on the contrary from Asymmetry as the expression of a holistic World.

Now, one has to be careful with such arguments, because you can easily fall prey to exactly the same formalism as the mathematicians.

Form is ***always*** a Consequence, and **never** a Cause!

Reality is asymmetric ***because*** it is the expression of a holistic World, where all things are mutually determining, and of different weightings. How could such a World ever by symmetric?

Causes in Reality are physical, chemical or biological. They may produce things which display certain Forms, but it is wholly wrong to ascribe their natures to any "pre-existing" forms. Remember, all Form in Reality is certainly temporary! This should tell us something!

In Ideality, of course, all Forms are eternal. This should tell us something else!

(3,686 words)