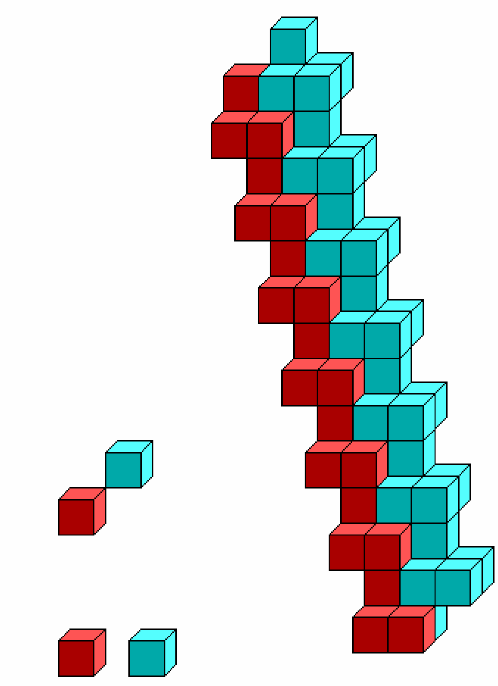
A Three Dimensional Problem

Diagrams, of course, play a vital role in communicating difficult, complex or involved areas to other people. Their role in this regard is unsurpassed, but they can also perform very effectively as tools for solving problems. Sometimes, without a kind of self-communication achieved by re-stating a problem diagrammatically, the researcher can be at a loss to even attempt a solution. Various contrasting atreas have come up in my experience over the last few years, and it is perhaps appropriate that a couple (at least) of these be laid out in this paper.

Perhaps the most significant of these cases was in the solution of a difficult problem in three dimensions, where a solid shape of some complexity had to be first tried out as a totally space-filling unit. That is one, like a perfect crystal that fills space with absolutely zero voids. These problems are difficult at the best of times, but in this particular case, I had spent a considerable amount of time trying to answer my problem by means of a series of hand-made models, but though I had established that the form could stack to fill space , I had not totally solved all the possible ways that this could be achieved. Suck-it-and-see methods had produced the single tessellation, but it was evident that more forms existed, and the total solution of the problem was required.

This case was to do with a self designed form which I had named the Soma Strand (after Piet Hein's "Soma delicious Soma").

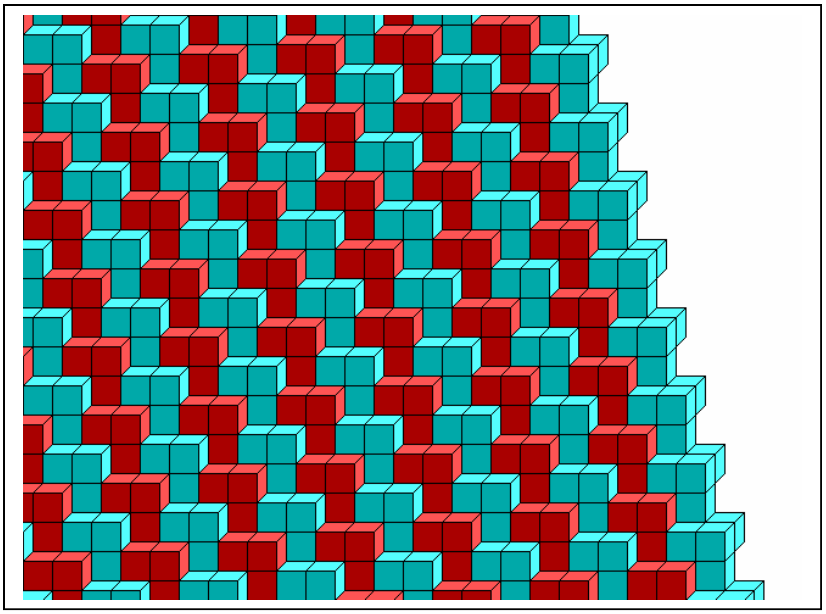
Long before the tessellation problem outlined above, I had to design the original form. This, in itself, took a great deal of time and model making, but what finally emerged was an infinite strand, with a re-entrant form, and congruent, singly re-entrant hexagonal faces (with 90^o and 270^o angles only). It soon became cleare (after making a bundle of these strands, that they definitely tessellated to fill three dimensional space completely. Careful model-making had established this exciting property, but, it must be said that cardboard, glue and bits of wooden dowling (without which the strands were impossible to construct) are not the ideal materials to facilitate detailed studies in the area of volume filling stacking, particularly of such difficult (and indeed infinite) strands. Attempts at solving the problem using 3D graphics packages were soon abandoned, as these so-called tools, may deal with three dimensions, but are generally NOT designed for meaningful and revealing visualisations essential to the designer and creator of new things. You just couldn't see what you were doing, and the figures soon became unintelligible. In addition to the complexity, that package had not helped me to adjust units effectively and lock them into the required precise positionings. Attempts at colouring to get some order out of the chaos only led to an obscuring of one part of the figure by another. I knew exactly what I wanted to do, but those facilities were simply not available.



As a last resort, I went all the way back to an ancient (and primitive) 2D drawing package that I had used for many years (***De Luxe Paint 2***). I had found this piece of software invaluable over many years in my studies (in 2D) into re-entrant polygons and their various tilings. I decided to try my hand at creating the tool that I needed WITHIN this old package. What I needed was a sort of "isometric" 3D system (with NO perspective as such) - where ALL lines were drawn exactly parallel to all others in the same direction. Now, of course, that it nothing special for lines in the plane of the screen, but what about the third dimension? These were all to be drawn in exactly the same direction – no meeting at some vanishing point. To make the system work for me I had to construct my Soma Strands with all faces either in the plane of the screen, or at 90^o to it, and these latter would all be drawn exactly parallel.

The figure alongside shows the results.

Another feature of the pack was also seen as a significant aid particularly in this pseudo 3D mode. This was the "lock" feature! I could set up a background "grid" to a given unit step size, and all movements across the screen could only be in integral units of this structure. Moves were integral steps, and all I had to do was make sure that my figures were draw so that they fitted perfectly into this system. Individual units could be moved as required and then just "locked" into place at exactly the correct positions without difficulty. Another feature that I was able to use was the "brush" feature. Any section of the screen could be chosen to use subsequently as a brush. This mean that there was no dot at the end of my cursor, but a complete figure, that I could move about and glue into place with a click of the mouse. This invaluable facility allowed me to start at the BACK of my conceived of figure, and build up a layer (using the brush) to overlap where necessary and give a very effective looking rack of strands as a result



Obviously, I had from the outset decided on a shading system for each strand) and a n alternating colouring system for the series of racks.

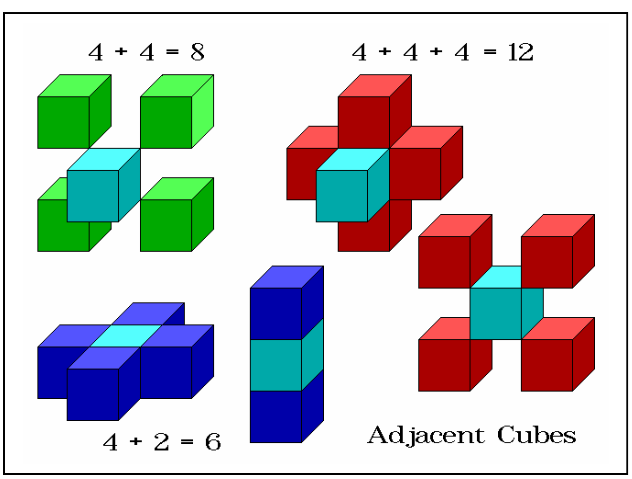
Also I had to choose my third dimension direction carefully so that locking could not only occur with the plane of the screen, but also, and crucially, in the direction INTO the screen. Strands could be positioned exactly in an x,y plane, and in the z direction with ease and certainty.

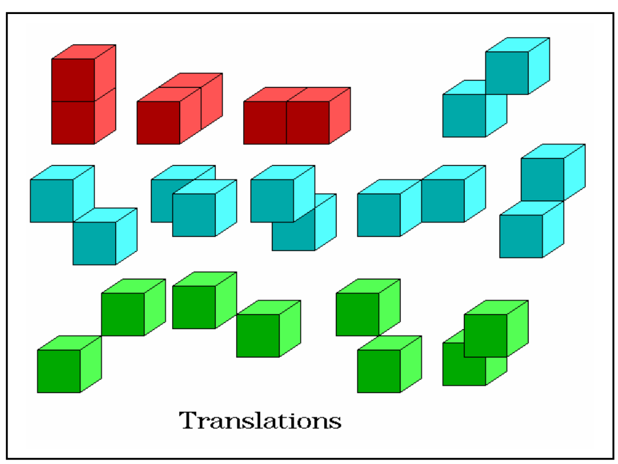
Now, after a few tries, it became evident that this system would work perfectly with the material I needed ti handle and study. Only three directions were ever in evidence, as all the angles in the Soma Strand were in combinations of 90o. The three directions were therefore those of the three axes in normal co-ordintes geometry. The first "rack" of a series of strands was soon produced with great clarity, and with the alternate colouring system revealed the form as self evident.

Next we had to answer the question, "Could other "racks" be produced and would all of these "stack" to fill space?"

The trial of my fictitious perspective was now put to a much tougher test.

Before I could proceed, however, I had to answer some fundamental questions about how unit cubes could be moved from one strand to an identical one alongside, that both left NO gaps between them AND allowed of no illegitimate "collisions". This problem was another task for our fictitious perspective system, and all possible translations were drawn out before any further trials were attempted.





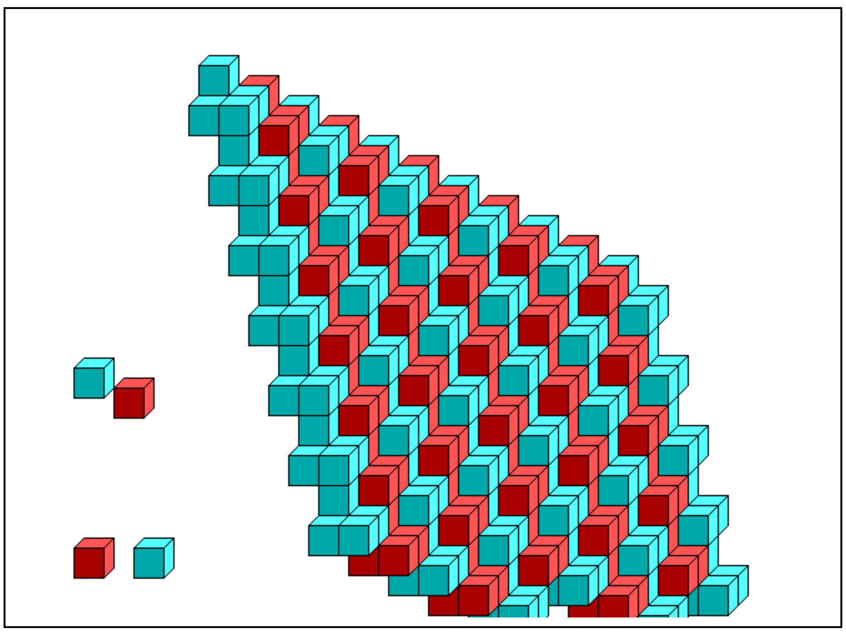
The results were that 13 such translations were possible:-

3 involved moving a square **face** onto an identical square face.
6 involved moving an **edge** onto an identical edge
4 involved moving a **vertex** onto an identical vertex

> A Note on Directions: I must comment on the 13 cases referred to here. Elsewhere I have been involved in designing pedagogical aids for the teaching of Rudolf Laban's system of orientations around an individual dancer. Instead of the usual scientific approach of using x, y, z coordinate geometry, Laban chose an intuitive system of directions with labels such as High, Deep, Front, Back, Left and Right, and a further extention to secondary and tertiary directions derived as intermediaries between the six named above. As you might guess, he ended up with precisely the same 13 orientations (26 if both directions are necessary, as he required) as we see here. The three orders of direction possessed different symmetries Primaries – 4 point, secondaries – 2 point, and tertiaries – 3 point. Interesting that his intuitive categorisations were in fact soundly based on real physical properties, and indeed conveyed a great deal more tha qualitiless coordinates.

These possibilities guyided the construction using our now trusted system to discover all possible "rackings" of the Soma Strand. The principle of collisions (mentioned earlier) was necessary for these trials to be addressed properly. This principle stated that if the given translation moved a cube from the first strand into the same space as another member of that strand, the racking was illegal and must be rejected. The principle also, of course, guided decisions in finding the subsequent "stackings"
Using our system and applying this simple principle enabled us to find all possible "rackings". There were exactly FIVE rackings, characterised by the actual translations that produced them.. Using the label V for vertex-to-vertex, and E for edge-to-edge, we were able to name the successful racks as

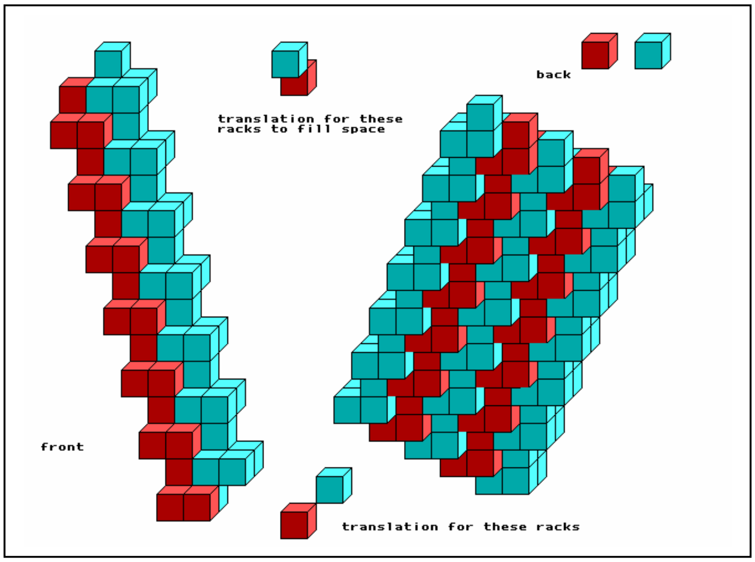
V_1 V_2 V_3 E_1 and E_2



Another "rack"

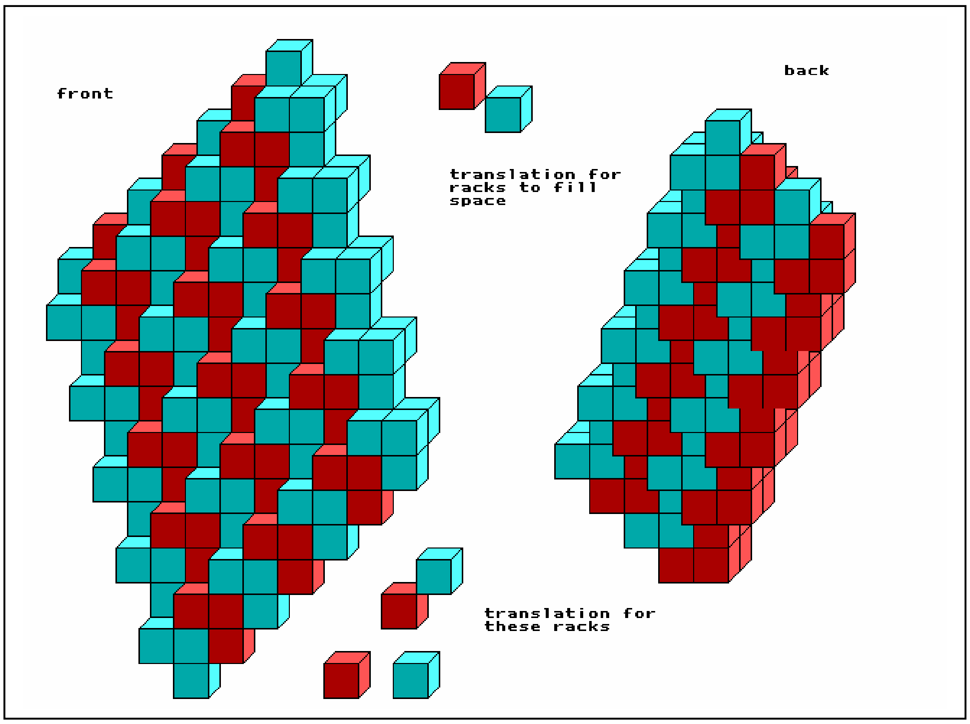
Which type of racking would this be, V or E?

Finally, the real douzy! Could we solve the problem of "stacking" our "racks" to fill space? Our efforts produced the following results:-



V_1 and E_2 works
V_1 and V_2 fails
V_2 and E_1 works
V_3 no partner fails

What had seemed to be an impossible problem was solved by a mixture of a fictitious perspective system and good diagramming.



NOTE: Remember, this section is not to teach the full content displayed. It can't do that because the full text would be significantly larger than this. It is merely to draw attention to diagramming techniques. Finally, on this topic, can the reader imagine an exposition of these interesting strands, racks and stacks WITHOUT quality diagrams?

NOTE: Anyone interested in the full story can find it in my pamphlet The Soma Strand. JS

(1596 words)